Shape Memory Alloy Temperature Compensation for Resonators

B. F. Keats, R. R. Mansour and R. B. Gorbet

University of Waterloo,

Waterloo, Ontario, Canada

Abstract — **In this paper we present a method for designing temperature compensated cavity resonators using shape memory alloys (SMA's). This paper gives a formula for the temperature drift of resonant frequency, which is valid for any conductor-loaded cavity regardless of its shape. This formula is combined with a field perturbation model and used to derive mechanical design constraints from a temperature drift constraint. Experimental results are given that confirm the feasibility of the proposed design approach.**

I. INTRODUCTION

Temperature drift is an important consideration when designing RF filters. To minimize drift, expensive high-density materials such as invar are used to construct resonators. Other materials such as aluminum have more favorable electrical properties and lower density, but cannot be used because of high temperature drift.

The unique properties of shape memory alloys (SMA) can be used to construct a tuning rod such that the field perturbation is dependant on temperature. The constraints on the temperature drift of a resonator can be used to derive constraints on the mechanical temperature response of an SMA tuning rod.

In this paper we propose using a spring biased SMA actuator spring to move a tuning rod inside a resonant cavity. This design can provide the field perturbation required to compensate for the effects of temperature drift. Also, since this design is thermally actuated, there is no added power consumption. It is then possible to reduce temperature drift using a passive system while adding little mass to the resonator.

II. SMA BEHAVIOR

Shape memory alloys (SMA) are materials that can revert to a memorized shape when heated above some threshold. For nickel-titanium alloys, this shape change is the result of a phase transformation from martensite to austenite. If and SMA compression spring is constructed, and some bias force is applied (using a spring or a mass) the spring will compress at low temperature. As the spring is heated, it will transform to the stiffer austenite phase, and revert to its original uncompressed shape.

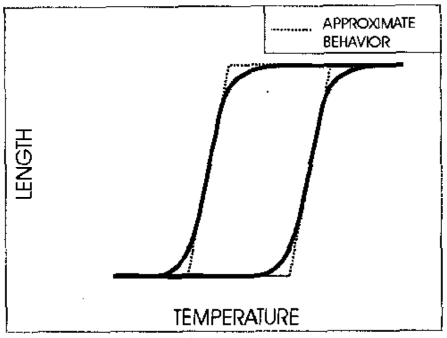


Fig. 1. SMA spring behavior

Fig. 1 shows the temperature response of an SMA spring with a bias force applied.

II. MODELING TEMPERATURE DRIFT

To derive a relationship between the resonant frequency of a cavity and temperature, a relationship between temperature and geometry must be established. For metals, which are allowed to expand freely, linear expansion can be assumed. The normal strain is:

$$\varepsilon_x = \varepsilon_y = \varepsilon_z = \alpha \Delta T \qquad (1)$$

where α is the coefficient of thermal expansion (CTE) and ΔT is the temperature change. The volume will expand, and the shear stain will be zero [1]. For a specific application the unconstrained expansion assumption must be evaluated on a case-by-case basis. Factors effecting this assumption will include temperature range and mounting.

A. Rectangular Resonators

Assuming free expansion for a conductor-loaded rectangular resonator with dimensions $b<a<d$, it can easily be shown that the resonant frequency after a rise in temperature ΔT is given by,

$$f(\Delta T) = \frac{c\sqrt{a_o^{\,2}(1+\alpha\Delta T)^2 + d_o^{\,2}(1+\alpha\Delta T)^2}}{2a_o d_o (1+\alpha\Delta T)^2} \qquad (2)$$

$$f(\Delta T) = f_o \frac{1}{(1+\alpha\Delta T)} \quad (3)$$

where, f_o is the resonant frequency of the unperturbed resonator.

It can be easily shown that these results are also valid for conductor-loaded cylindrical resonators. It is worthwhile to check the accuracy of the HFSS solution since we are dealing with an extremely small shift in resonator frequency. The simulated temperature drift results correlate extremely well with predicted results. The largest error in the predicted results is *0.0008%*.

B. Other Cavity Resonators

This temperature model (3) can be derived analytically for rectangular and cylindrical cavities. Simulations using HFSS suggest that this formula is valid for any conductor-loaded cavity of any shape.

Equation (3) was tested using a reentrant coaxial resonator (Fig. 2a) [2] a metal disk resonator (Fig. 2b) [3], and a half cut resonator (Fig. 2c) [4]. No support structure was included in these simulations for simplicity.

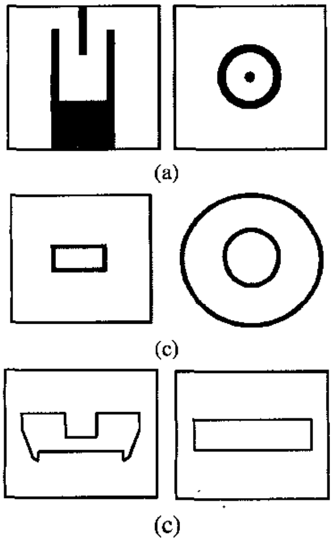


Fig. 2. (a) Reentrant coaxial resonator [2], (b) Disk resonator [2], (c) Half cut resonator [4].

The results of these simulations are shown in Fig. 3. Again, the HFSS simulations conform to equation (3) extremely well. Here the maximum error incurred is *0.0014%*.

The high correlation between the simulated and the predicted resonant frequencies for these resonators implies that the temperature model in equation (3) is valid for any conductor-loaded cavity that is allowed to expand freely regardless of its shape.

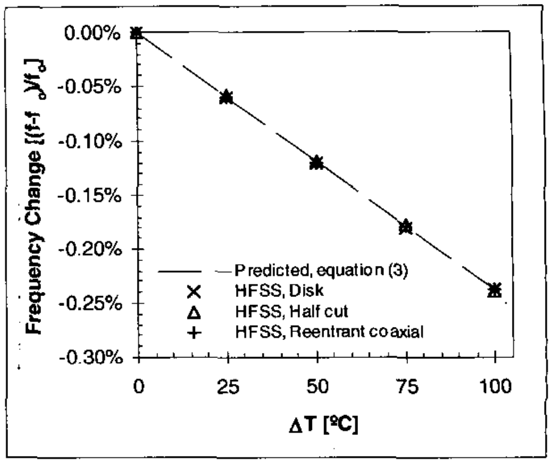


Fig. 3. Resonant frequency predictions for other resonators.

III. Ideal Tuning Rod Behavior

Using a field perturbation model and the expected temperature drift of an uncompensated resonator (3), an ideal tuning rod behavior can be derived. A tuning rod with this behavior will perfectly compensate for the temperature drift and result in a constant resonant frequency.

A modified version of the small perturbation model for a rectangular cavity is used [5].

$$\frac{f - f_o}{f_o} = -s\frac{\Delta V}{V_o} \quad (4)$$

The parameter s must be fit to simulated data for a particular resonator (s=2 for a rectangular cavity resonator). This model assumes that the perturbation response is linear, which is a reasonable assumption for the range of perturbation required for this application.

The base frequency f_o from the modified field perturbation model (4) will depend on temperature if the cavity is expanding. This is due to the fact that the size of the unperturbed cavity is changing with temperature. The change in the base frequency f_o is predicted by the temperature drift model (3). The volume of the unperturbed cavity V_o will also change with temperature.

If these results are substituted into the modified perturbation model (4), the resonant frequency of a cavity subjected to temperature changes and field perturbations can written as,

$$f = f_o \frac{1}{(1+\alpha\Delta T)}\left(1 - s\frac{\Delta V}{V_o(1+\alpha\Delta T)^3}\right) \quad (5)$$

In equation (5), the base frequency f_o corresponds to the resonant frequency of the cavity with no temperature change, and unperturbed by the tuning rod.

For perfect temperature compensation, the frequency f for the perturbed cavity subject to temperature changes (5) should be a constant f_d. Assuming that the tuning rod is a cylinder with a constant radius and variable length, the desired frequency is chosen as the resonant frequency of the unperturbed cavity at the maximum expected change in temperature ΔT_m.

$$f_d = f_o\left(\frac{1}{(1+\alpha\Delta T_m)}\right) \quad (6)$$

This choice for f_d means that the length of the tuning rod which perturbs the field will be zero at the maximum change in temperature ΔT_m. A variable tuning rod length $l_f(\Delta T)$, and the a constant radius r replace the tuning rod volume ΔV. By solving for $l_f(\Delta T)$ the ideal tuning rod behavior can be derived.

$$l_f(\Delta T) = \frac{V_o\alpha(\Delta T_m - \Delta T)(1+\alpha\Delta T)^3}{s\pi r^2(1+\alpha\Delta T_m)} \quad (7)$$

Equation (7) describes the desired behavior of the tuning rod that will compensate perfectly for temperature drift. The rod length l_f will be zero at the maximum temperature T_m. Note that since the coefficient of thermal expansion α will be small, the field length l_f will be approximately linear with respect to the change in temperature ΔT. By using a simple field perturbation model, it is possible to derive an analytical result, which is very convenient for design purposes. If a more accurate model is required, this same procedure can be followed using a polynomial field perturbation model, however the equation will need to be solved numerically.

IV. Mechanical Design Constraints

Since it is unlikely that a compensation mechanism can be designed to follow the ideal tuning rod behavior exactly, it would be useful to derive a range of behavior that will meet specified design constraints.

$$\delta = \frac{\Delta f}{\Delta T_m f_d 10^{-6}} \text{ [ppm/°C]} \quad (8)$$

The temperature drift in *ppm/°C* (8) can be specified for a given application. In order to find the design boundary where the drift δ is equal to the maximum temperature drift δ_m the desired frequency (6) must be modified.

$$f_d = f_o\left(\frac{1}{(1+\alpha\Delta T_m)}\right)\left(1 \pm \Delta T_m \delta_m 10^{-6}\right) \quad (9)$$

By repeating the pervious derivation, a limit on the temperature-length behavior of the tuning rod can be derived.

$$l_{fd} = l_f \pm \frac{V_o(1+\alpha\Delta T)\Delta T_m \delta_m 10^{-6}}{s\pi r^2(1+\alpha T_m)} \quad (10)$$

A design envelope for the mechanical response of the tuning rod is shown in Fig. 4. If the temperature response of a particular tuning rod design lies within this design envelope, then the temperature drift will meet the constraint used in equation (10)

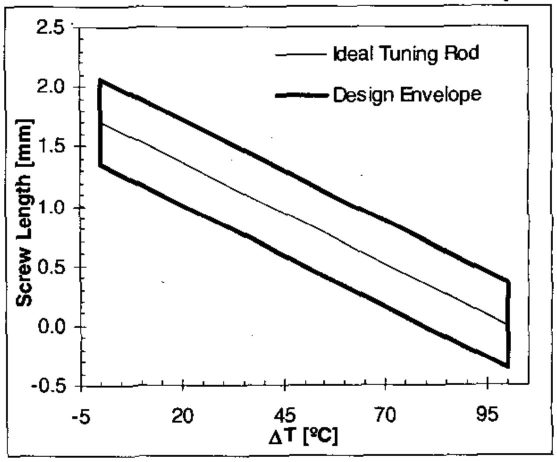


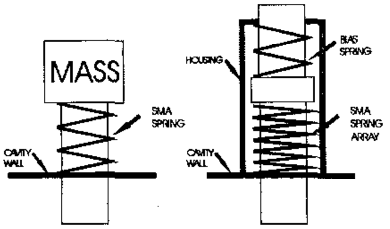
Fig. 4. Mechanical design envelope

With the use of Fig. 4, the mechanical design of the tuning rod can be divorced from the RF design. It also provides firm design specifications for the mechanical designer derived from temperature drift specifications.

V. Design and Results

Because of the relatively narrow actuation range of an SMA spring *(10-20°C)*, several SMA springs in series will be required to compensate over a useful temperature range. A diagram of the full design is shown in Fig. 5b, while the full design prototype is shown in Fig. 6.

In order to prove that the concept of temperature compensation using shape memory alloys is feasible, some preliminary experiments were performed. A mass biased tuning rod was designed (Fig. 5a).



(a) (b)

Fig. 5. (a) Test design, (b) Full tuning rod design

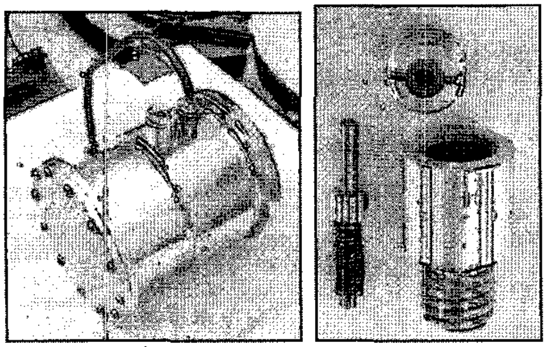


Fig. 6. SMA tuning rod prototype

This design uses one spring that actuates over a *15°C* temperature range when heated. It was heated in a kiln from *22°C* to *80°C*. The results, shown in Fig. 7, show excellent temperature compensation over the actuation range of the SMA.

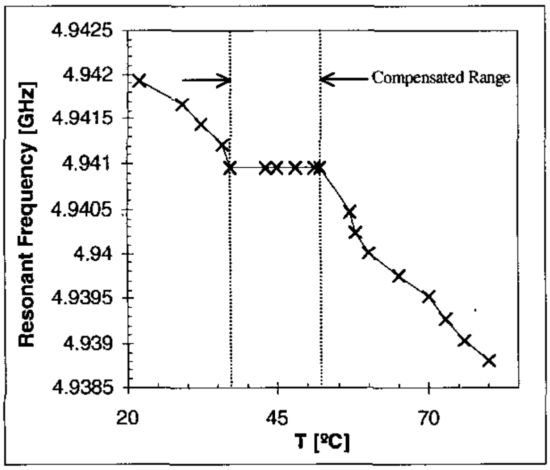


Fig. 7. Resonant frequency compensated by SMA design

It should be mentioned that this experiment did not measure the resonant frequency as the cavity cooled. Because of the hysteresis in the SMA behavior it will be important to evaluate a design with both heating and cooling. However, the hysteresis of the used SMA material is within the boundary outlined in Figure 4 and one would expect to get an overall temperature drift less than 2 ppm/C. The thermocouple used in this experiment is also only accurate to *±1°C*. This will add some uncertainty to the results.

This experiment shows that temperature compensation using SMA's of cavity resonators is indeed plausible. A full prototype using this design would incorporate several SMA springs in series. This would allow compensation over a broader temperature range than that shown in Fig. 7.

V. Conclusion

This paper presented an analytically derived model for the temperature drift of conductor-loaded cavity resonators. This result was used to form a comprehensive method for designing SMA tuning rods capable of compensating for temperature drift. Experimental results were shown demonstrating that the concept of SMA temperature compensation is feasible.

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